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An Accurate Multiphase Upscaling for Flow and Transport in Heterogeneous Porous Media

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Abstract

Subgrid effects can have a strong influence on flow and transport in oil reservoirs. In this work a new model for the representation of subgrid terms is introduced and applied to two-phase reservoir flows. The model entails a construction of pseudo relative permeabilities using accurate inlet boundary conditions for the saturation field. Motivation for the form of the model is provided through a consideration of asymptotic techniques for two-phase flow. The numerical computation of the subgrid terms and the implementation of the overall method are described. The accuracy of the new subgrid representation is compared to that of coarse scale models with no subgrid treatment and to coarse models based on traditional pseudo relative permeabilities. In essentially all cases the new model provides more accurate coarse scale predictions relative to reference fine scale results. These are preliminary results.

1. Introduction

The effects of reservoir heterogeneity at a scale smaller than a typical simulation grid block can have a significant impact on reservoir flow. A number of different approaches for the modeling of these subgrid effects have been introduced. The most commonly applied methods involve the use of pseudo relative permeabilities and the use of specialized coarse gridding procedures. Though effective in many cases, both of these approaches are known to suffer from some drawbacks, including a potentially high level of process-dependency (in the case of pseudo relative permeabilities) and the inability to provide very high degrees of coarsening (in the case of gridding procedures).

As indicated above, one of the approaches for upscaling includes the use of pseudo relative permeabilities. There have been a number of previous papers addressing the development and evaluation of methods based on pseudo relative permeabilities. Barker and coworkers [1] and Darman et al.[4] discussed and evaluated a number of the relevant methods. In most cases, these methods differ from one another in the way in which the reference fine scale results are post-processed to generate the upscaled or pseudo relative permeability curves. In many cases, a more critical issue is the boundary conditions applied to the local fine scale problem used to compute the upscaled functions. Recent papers by Wallstrom et al. [14,15] address this issue via the development of "effective flux" boundary conditions. These boundary conditions attempt to mimic the average effects of the large scale flow on the local problem and to correct the bias inherent in standard approaches, which tend to overestimate the impact of local high permeability regions in the calculation of the upscaled functions. Other approaches for upscaling include the use of flow-based grid generation procedures, such as nonuniform coarsening, [7] and the use of multiscale methods, such as multiscale finite element methods [12]. For transport problems, these methods in essence attempt either to minimize subgrid effects through use of an "optimal" grid (grid generation techniques) or to reconstruct the full fine grid velocity field using the subgrid representation (multiscale methods). Though both approaches are effective in many cases, they do have some limitations. Methods that require velocity reconstruction, as currently implemented, rely on the use of two grids, which can lead to complications in data structures and the mapping of variables between grids.

In our paper, we present an approach that uses the single-phase flow information for the inlet saturation boundary conditions in pseudo relative permeabilities. In particular, using single-phase time of flight information, the inlet boundary conditions are computed. The proposed approach relies on recent asymptotic analysis (see [11]), where the authors show that the single-phase flow can have dominant effect on two-phase flow simulations. From this asymptotic analysis, it follows that the saturation is a smooth function of single-phase time of flight and, thus, it can be approximated via a linear function of time of flight. This result suggests that the use of single-phase flow based grid as in [7] can make pseudo computations more accurate. However, in this paper, we present pseudo relative permeability computations in a general coarse grid using time of flight function. The main difficulty of the proposed approach is to set the dynamic change of the function that depends on time of flight. This is done through simplified linear time dependent functions with an adjustable parameter. We are currently considering various options where one can avoid the adjustable parameter. These results will be reported elsewhere.

This paper proceeds as follows. We first present the fine scale equations and then discuss pseudo relative permeability approach. In the following section, we discuss the improved pseudo relative permeability computations and the motivation. A few preliminary numerical results are presented in the last section.

2. Fine Scale Equations for Two-Phase Flow

Our goal is the development of accurate subgrid models for two-phase (oil-water) flow problems. In this work we will focus on systems that are convection dominated on the fine scale and will therefore neglect the effects of capillary pressure, gravity and compressibility. We will further assume that porosity is constant. This system can be represented in dimensionless form via the pressure and saturation equations:

$$\nabla \cdot (\lambda(S)\mathbf{k} \cdot \nabla p) = 0, \qquad (1)$$

$$\frac{\partial S}{\partial t} + \nabla \cdot \mathbf{F} = 0, \qquad (2)$$

where $\lambda(S)$ is the total mobility, S is the water saturation, $\mathbf{k}$ is the permeability tensor, here taken to be diagonal, p is pressure, and $\mathbf{F}$ is the fine scale convective flux function. The total mobility is given by

$$\lambda(S) = \lambda_w + \lambda_o = \frac{k_{rw}}{\mu_w} + \frac{k_{ro}}{\mu_o}, \qquad (3)$$

where the subscript j designates the phase ($j = o, w$, with o indicating oil and w water), k_{rj} is the relative permeability to phase j and μ_j is the viscosity of phase j. The flux function $\mathbf{F}$ is given by $\mathbf{F} = \mathbf{v} f(S)$, with

$$f(S) = \frac{k_{rw}/\mu_w}{k_{rw}/\mu_w + k_{ro}/\mu_o}, \qquad (4)$$

and the total velocity $\mathbf{v}$ expressed as:

$$\mathbf{v} = \mathbf{v}_w + \mathbf{v}_o = -\lambda(S)\mathbf{k} \cdot \nabla p. \qquad (5)$$

In this work we assume that the fine scale system is characterized by a single set of relative permeability curves. This assumption is introduced only for simplicity, as our method can handle cases involving multiple sets of fine scale curves.

3. Pseudo Relative Permeability Models.

The coarse scale description of the oil-water system is again expressed in terms of a pressure equation and a saturation equation. Standard coarse scale representations involve the use of equations of the same forms as Eqs. 1 and 2 but with modified (coarse scale) parameters, as follows:

$$\nabla \cdot (\lambda^*(\mathbf{x}, \overline{S})\,\mathbf{k}^* \cdot \nabla \overline{p}) = 0, \qquad (6)$$

$$\frac{\partial \overline{S}}{\partial t} + \nabla \cdot \mathbf{F}^*(\mathbf{x}, \overline{S}) = 0. \qquad (7)$$

In these and subsequent equations we use the overbar to designate coarse grid pressure and saturation. These dependent variables can be thought of as volume averaged fine grid quantities, where the volume average is over the region corresponding to the coarse grid block. We use the * superscript to indicate an upscaled (effective, equivalent or pseudo) function, which will in general be computed through the solution of a local fine grid problem. These functions also depend on spatial location $\mathbf{x}$ since they may differ in each coarse scale block (even when the fine scale functions are the same throughout the model).

In the approaches based on pseudo relative permeabilities (cf. [1, 4]) a fine scale problem is solved and coarse scale mobility ($\lambda_i^*(\overline{S})$) and fractional flow functions ($f_i^*(\overline{S})$) are computed (the i subscripts indicate that these functions may depend on direction). From these two functions, pseudo relative permeabilities $(k_{rj}^*)_i$ can be determined via generalizations of the fine scale relations between f, λ and k_{rj} given in Eqs. 3 and 4. The quantity $\mathbf{F}^*$ can then be expressed via $F_i^* = \overline{v}_i f_i^*(\mathbf{x}, \overline{S})$. A number of numerical procedures are also available for the calculation of the equivalent grid block permeability tensor $\mathbf{k}^*$ [6, 16, 13, 17]. In our numerical simulations, we employ global approaches to compute $\mathbf{k}^*$. The coarse scale representation given by Eqs. 6 and 7 has the advantage that it is of the same form as the fine scale model and is therefore immediately applicable for use in current simulators.

4. Improved Pseudo Relative Permeability Model

To achieve high accuracy in the coarse-scale simulations, the inlet boundary conditions for the saturation need to be adapted to the global flow directions. We propose the use of time of flight associated with single-phase flow in imposing the inlet boundary conditions. Our motivation stems from an asymptotic analysis presented in [11]. First, we briefly mention some analytical results from [11]. Denote the initial stream function and pressure by $\eta = \psi(\mathbf{x}, t=0)$ and $\zeta = p(\mathbf{x}, t=0)$. Then the equation for the pressure can be written in the (η, ζ) coordinate system as

$$\frac{\partial}{\partial \eta}\left(|k|^2 \lambda(S)\frac{\partial p}{\partial \eta}\right)+\frac{\partial}{\partial \zeta}\left(\lambda(S)\frac{\partial p}{\partial \zeta}\right)=0, \qquad (8)$$

where $\mathbf{k} = k\mathbf{I}$, $\mathbf{I}$ is the identity matrix. For simplicity, $S = 0$ at time zero is assumed. We consider a typical boundary condition that gives high flow within the channel, such that the high flow channel will be mapped into a large slab in the (η, ζ) coordinate system. If the permeability variation within the channel (in the η direction) is weak, the saturation within the channel will depend on ζ. In this case, the leading order pressure $\hat{p}$ will depend only on ζ, and it was shown ([11]) that

$$p(\eta, \zeta, t) = \hat{p}(\zeta, t) + \text{high order terms}, \qquad (9)$$

where $\hat{p}(\zeta, t)$ is the dominant pressure.

This asymptotic expansion shows that the pressure (which varies in time due to saturation effects) depends strongly on the initial pressure ζ; i.e., the leading order term in the asymptotic expansion is a function of initial pressure and time only. This initial pressure contains global information. From Eq. 9, it follows that

$$\mathbf{v} = C^*(\mathbf{x}, t)\mathbf{v}_0, \qquad (10)$$

where $\mathbf{v}_0$ is single-phase flow velocity field, $\mathbf{v}$ is two-phase flow velocity field, and C^* is a smooth (coarse-scale) function. Next, we introduce time of flight function τ

$$\mathbf{v}_0 \cdot \nabla \tau = 1. \qquad (11)$$

The saturation equation for two-phase transport can be written in terms of τ in the following way:

$$\frac{\partial S}{\partial t} + C^*(\mathbf{x}, t)\frac{\partial S}{\partial \tau} = 0. \qquad (12)$$

This equation suggests that S is a smooth function of τ away from interfaces which can be tracked separately. This is an underlying idea of the proposed approach. Because S is a smooth function in terms of τ, we propose to approximate the inlet boundary conditions for S using a linear function in τ. More precisely, S at the inlet can be described by

$$S|_{\text{inlet}} = D_0\tau + D_1, \qquad (13)$$

where D_0 and D_1 are generic constants which are generally time-dependent functions. This is a first order approximation.

Eq. 12 suggests that one can perform pseudo computations using flow based grids. However, our objective in this paper is to set boundary conditions in a general grids using global information from single-phase flow simulations. Moreover, we would like to keep pseudo relative permeability computations local, thus, *apriori* determine the constants in Eq. 13. In our preliminary computations, we consider the following inlet boundary conditions:

$$S|_{\text{inlet}} (\mathbf{x}, t) = \min(A_1 t - A_0\tau + B, 1), \qquad (14)$$

where we have re-named the constants (cf. Eq. 13) in order to show the explicit time dependence. Here, t is the local dimensionless time (PVI, see the definition below) and A_0 and B are chosen such that $S|_{\text{inlet}}$ is one at the minimum of τ, zero at the maximum of τ at (local) time zero. Note that at the minimum of τ, S must be large, and therefore, there is a negative sign before A_0. In particular, $A_0 = \frac{1}{\tau_{max} - \tau_{min}}$ and $B = \frac{\tau_{max}}{\tau_{max} - \tau_{min}}$, where τ_{min} and τ_{max} are minimum and maximum values of τ along the inlet edge. The term $A_1 t_{pvi}$ increases and $S|_{\text{inlet}}$ finally reaches 1 at the inlet. If A_1 is small, then the local saturation reaches steady state before any large changes occur at the inlet. For very large values of A_1, the inlet boundary conditions reaches 1 quickly and the results are similar to standard pseudo relative permeability computations. In our simulations, we find that the results of the proposed approach depend on the values of A_1 and test various values of A_1. We are currently considering various options where one can avoid adjusting A_1.

For the computation of global flow and transport, we do not modify $\lambda(S)$ and only modify $\mathbf{F}^*(S)$ using traditional pseudo approaches with inlet saturation evolution according to Eq. 14. This is consistent with out asymptotic results. Similarly, in [10, 9], the authors do not modify the total mobility in upscaled computations.

Since the global single-phase simulations are performed, we use them for the computations of upscaled permeability and the computations of the local problems. We would like to note that our approach is a slight modification of existing pseudo relative permeability approach, and thus also applicable to other similar approaches, e.g., generalized convection-diffusion approaches [8]. One can use single-phase flow based information in imposing saturation boundary conditions in these approaches.

5. Numerical results

In this section we present simulation results for several two dimensional systems. We consider reservoir cross sections characterized by two point statistics as well as more complex geological models. In all cases the coarse models are generated from the fine models through a uniform coarsening. Simulation results are presented for oil cut as a function of pore volume injected (PVI). Oil cut is given by q_o/q_t. Here the total flow rate q_t is given by $q_t = q_o + q_w$, where q_o and q_w are the flow rates of oil and water at the production edge of the model. Pore volume injected, defined as $\frac{1}{V_p}\int_0^t q_t(\tau)d\tau$, with V_p the total pore volume of the system, provides the dimensionless time for the displacement. Results are presented for the fine model, the coarsened model with single-phase upscaling, the coarsened model using pseudo relative permeabilities, and the coarsened system using our modified pseudo relative permeabilities. In all cases, the relative permeabilities are taken to be $k_{rw}(S_w) = S_w^2$

and $k_{ro}(S_o) = S_o^2$. We consider simulations with global boundary conditions that are fixed in time and defined as $p=1$ at $x=0$, $p=0$ at $x=1$ and no flow boundary conditions on lateral boundaries, unless otherwise is specified.
We compute the upscaled grid block permeabilities $\mathbf{k}^*$ using a global solution method. This entails the global solution of the flow subject to constant pressure - no flow boundary conditions in each coordinate direction.
The numerical methods used for the fine and coarse solutions are as described above for the calculation of the coarse scale functions (e.g., TVD solution of the saturation equation). In calculations involving pseudo-relative permeabilities, however, we also apply a second-order method for the solution of the saturation equation. Directional pseudo functions for $\mathbf{F}^*$ and λ^* are computed based on local two-phase flow solutions.
Our examples involve a square domain in the $x-z$ coordinate system. The fine scale model is of dimension 100×100 and the uniform coarse scale models are 10×10. The fine scale geological descriptions are geostatistical realizations of unconditioned, log-normally distributed permeability fields with prescribed correlation structure and variance (σ^2) in $\log k$. The correlation structure is specified in terms of dimensionless correlation lengths in the x and z -directions, l_x and l_z, nondimensionalized by the system length. The realizations were generated using GSLIB algorithms [5]; in all cases a spherical variogram model was used. We also consider channelized permeability fields using SPE Comparative Solution Project [3]. In this case, we interpolate the permeability field onto 100×100 fine grid. This interpolation preserves the fine-scale features of the permeability field. As we mentioned earlier, our results are very preliminary and we restrict ourselves to only a few test examples. The proposed method and its modifications are currently under investigation.
The permeability statistics of our first example are specified as $l_x = 0.2$, $l_z = 0.05$ and $\sigma^2 = 1$. Results for oil cut versus PVI for flow is shown in Figure 1. In these and subsequent figures, the solid curves represents the fine model, the dashed-dotted curve represents the coarsened model with single-phase upscaling, the dotted curve the coarsened model using pseudo relative permeabilities, and the dashed curve the coarsened model with modified pseudo relative permeabilities. In this example, the value of A_1 is taken to be 1.8. We observe from this figure that the modified pseudo provides an accurate approximation of reference solution. In particular, the relative L_2 errors of oil cut are 13% for single-phase coarse, 8% for pseudo, and 4% for modified pseudo approach.
In the next numerical example, we consider a permeability field with long correlation length in horizontal direction. In particular, we choose $l_x = 0.4$, $l_z = 0.05$ and $\sigma^2 = 1$. In Figure 2, we plot the fine-scale reference oil cut, oil cut corresponding to pseudos and modified pseudos. Since our goal is to improve pseudos, we do not present oil cut for single-phase coarse approach which is typically worse than the pseudo approach. We observe from this figure that modified pseudo approach provides an accurate approximation of fine-scale reference oil cut curve. In particular, the relative L_2 errors of oil cut are 14% for pseudo, and 3.6% for modified pseudo.
Finally, we present numerical results for a layer 44 of SPE Comparative Solution Project [3]. We consider both flow from left to right and bottom to top. In the case of the flow from bottom to top, $p=1$ at $y=0$, $p=0$ at $y=1$ and no flow boundary conditions on lateral boundaries. The numerical results are presented in Figure 3 and Figure 4. In these numerical results different values of A_1 is used as indicated in the caption of this figures. We see from these numerical results that the optimal value of A_1 is case dependent, though one can achieve high accuracy with an appropriate choice of A_1. Currently, we are considering various replacements for $A_1 t$ in the formula for inlet saturation in order to avoid adjustable parameters.

6. Conclusions

In this paper, a new model for the representation of subgrid terms within pseudo relative permeability approach is introduced and applied to two-phase reservoir flows. Motivation for the form of the model is provided through a consideration of asymptotic techniques for two-phase flow. Our model has an adjustable parameter. The accuracy of the new subgrid representation is compared to that of coarse scale models based on pseudo relative permeabilities. In all cases the new model provides more accurate coarse scale predictions relative to reference fine scale results. These are preliminary results.

7. Acknowledgments

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Nomenclature

$\mathbf{F}$ flux function
$\mathbf{k}$ permeability tensor
k_r relative permeability
l correlation length
p pressure
S saturation
t time
τ time of flight
$\mathbf{v}$ velocity
V_p pore volume
x spatial location
λ mobility
μ viscosity
ζ single-phase pressure
η single-phase stream-function

ψ	stream-function
C^*	a scalar function
A	pseudo defining parameter
$\mathbf{v_0}$	single-phase velocity field
σ^2	variance of log of permeability

Subscripts

j	phase
o	oil
w	water

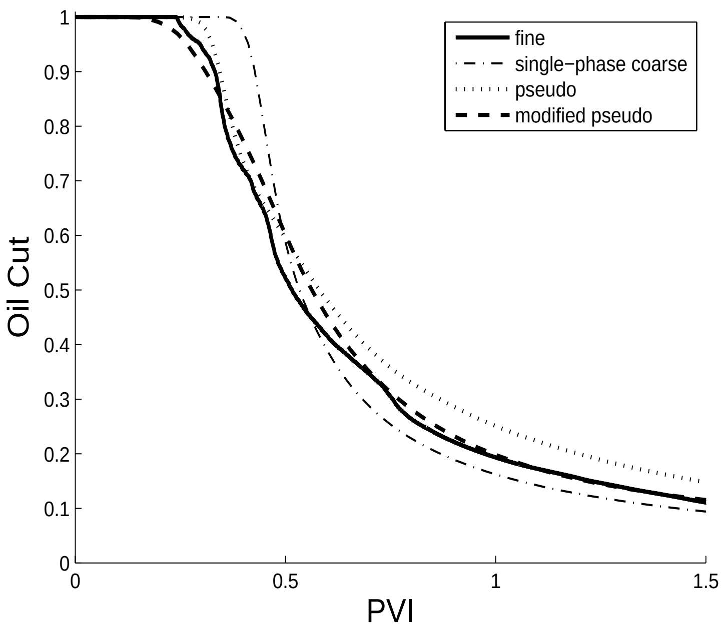

Figure 1. $l_x = 0.2$, $l_z = 0.05$, $\sigma = 1$, $A_1 = 1.8$.

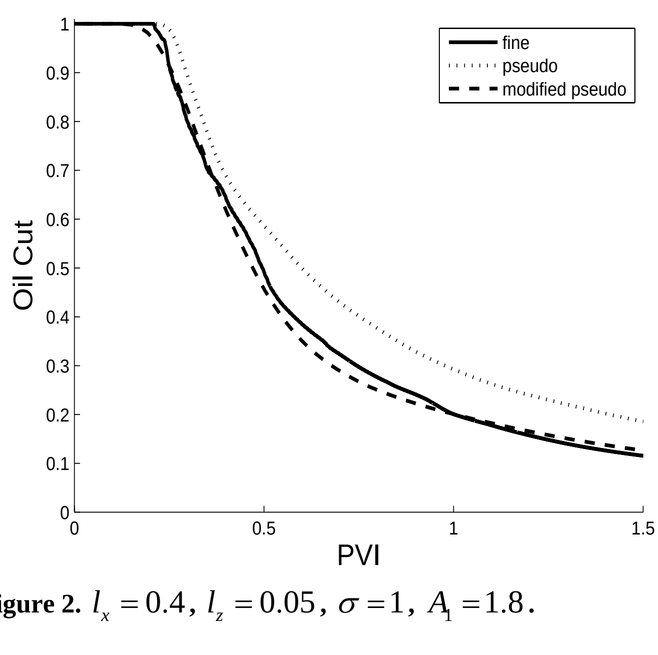

Figure 2. $l_x = 0.4$, $l_z = 0.05$, $\sigma = 1$, $A_1 = 1.8$.

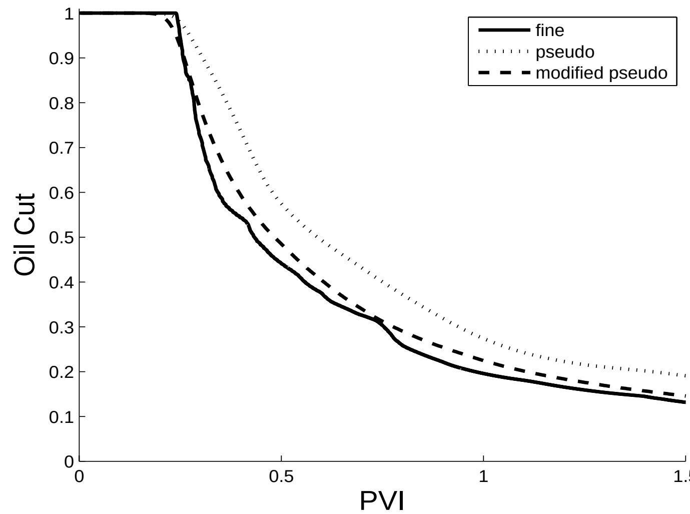

Figure 3. Layer 44 of SPE 10. $A_1 = 0.01$. Flow in horizontal direction.

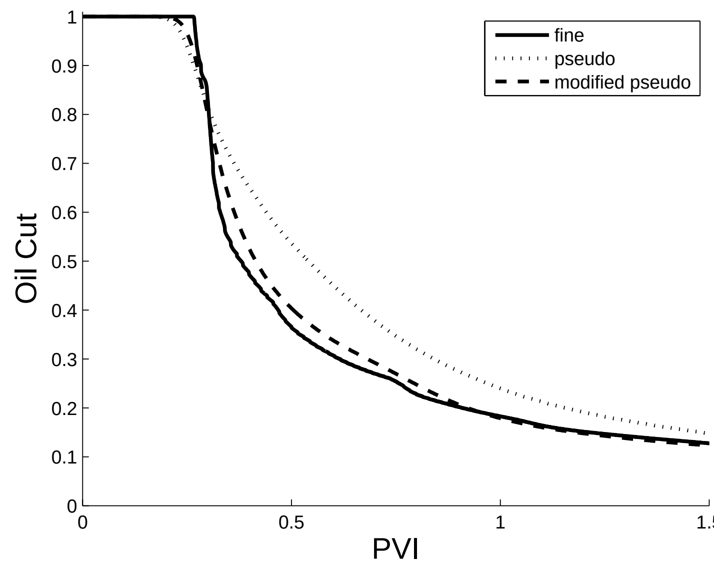

Figure 4. Layer 44 of SPE 10. $A_1 = 0.5$. Flow in vertical direction.

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